

Lord Kelvin and the Age of the Earth

Introduction

It is widely believed that in 1862 Lord Kelvin, using the laws of thermodynamics describing the rate at which a hot iron ball cooled, 'proved' that the Earth could not be much more than about 100 million years old, thus putting physics at odds with the nascent sciences of geology and evolutionary biology. It is also widely believed that Kelvin was wrong because he was unaware of the phenomenon of radioactivity which was not discovered for another 30 years. Both of these beliefs are false. Kelvin's arguments were far more sophisticated than a simple application of a law of cooling and it turns out that radioactivity plays a very minor role in the energy budget of the Earth. The confusion begins with a short paragraph in Newton's Principia.

Newton's estimate of the age of the Earth

In book III, halfway in a long chapter on comets, there is this paragraph:

... for a globe of iron of an inch in diameter, exposed red-hot to the open air, will scarcely lose all its heat in an hour's time, but a greater globe would retain its heat longer in the proportion of its diameter; because the surface (in proportion to which it is cooled by the contact of the ambient air) is in that proportion less in respect of the quantity of the included hot matter; and therefore a globe of red-hot iron equal to our earth, that is about 40,000,000 feet in diameter, would scarcely cool in an equal number of days, or in above 50,000 years.

To recast this argument in modern language, we would say that, since the ratio of the volume of a sphere to its surface area is proportional to the diameter of the sphere, if it takes 1 day for a small red-hot iron sphere one inch in diameter to cool, it will take 480,000,000 days for a sphere the size of the Earth to cool.

Now 480,000,000 days is actually 1.3 million years, not 50,000. But Newton's error (if that is what it is) is not important. What he is saying is that the Earth cannot be infinitely old and that it should be possible to calculate its age using experiment and logical deduction.

By 1862 when Lord Kelvin published his famous estimate, the laws which governed the cooling of hot objects in empty space were still not known. Stefan's law which predicted that a black body would radiate heat in accordance with the fourth power of its absolute temperature was not published until 1877. It is, however, interesting none the less to calculate the time it would take a red-hot iron ball the size of the Earth to cool down assuming that it is a black body obeying Stefan's law. To keep things simple we shall assume that iron has infinite conductivity i.e. it is always at the same temperature T throughout. Basically we have:

$$MS \frac{dT}{dt} = -\sigma AT^4 \quad (1)$$

where M is the mass of the sphere, S is the specific heat capacity of iron, σ is Stefan's constant and A is the surface area of the sphere. This leads to

$$\frac{dt}{dT} = -\frac{RS\rho}{3\sigma T^4} \quad (2)$$

where R is the radius of the Earth and ρ is its density.

Integrating this expression with respect to T gives us

$$t = -\frac{RS\rho}{9\sigma} \left(\frac{1}{T^3} - \frac{1}{T_0^3} \right) \quad (3)$$

Now the initial temperature T_0 is much higher than the final temperature T so we can safely ignore this term¹. Putting in realistic data for the constants² we can deduce that the time t taken to cool from a high temperature to the present day temperature of around 300 K is $4.4 \times 10^{19} / 300^3$ or 1.6×10^{12} s. This equates to 51,000 years which, by a happy coincidence, is almost exactly Newton's original guess!

Kelvin's calculation

As I have said, in 1862 Kelvin did not know of Stefan's law so he was unable to perform the above calculation – which was, perhaps, a good thing! Instead he drew inspiration from another piece of mathematics recently worked out by Fourier concerning the conduction of heat through solid bodies. Using Fourier's theory, one could calculate how heat would distribute itself through a solid body by conduction. At the heart of the theory was the simple idea that the rate of transfer of heat through a small cross section would be proportional to the temperature gradient in the same way that the electric current through a wire is proportional to the voltage across it.

Kelvin asked himself, not how long would it cool a hot sphere to cool to a surface temperature of 300K, but how long would it take for a sphere of rock, initially at a temperature of T_0 to cool to the point where the *temperature gradient* at the surface was equal to its present value of about 4 degrees K per 100 m (0.04 K/m). His basic assumption was that initially the Earth was at a uniform temperature of 3900 K (the melting point of rock) except at its surface which is maintained (presumably by radiation into space) at a constant temperature of 300K³. Obviously this implies that, initially at least, the temperature gradient at the surface would be infinitely large and consequently the Earth would lose heat at an infinite rate. This state of affairs does not last, however, as heat can only be supplied to the surface by (he assumed) conduction from the interior. Fourier's theory predicts that the graph of temperature across the globe will decay exponentially and that the temperature gradient at the surface will gradually fall.

The graph below is taken from Kelvin's 1862 paper 'On the Secular Cooling of the Earth'. It has been rotated 90° to put the independent variable (the depth below the surface) on the X axis. My annotations are in colour. It should be noted that the graph only shows the top 5% of the Earth's radius. What this means is that, according to Kelvin, the Earth's interior has barely cooled at all throughout its history.

The formula that Kelvin used to calculate the time t was

$$t = \frac{\rho S T_0^2}{\pi k \beta^2} \quad (4)$$

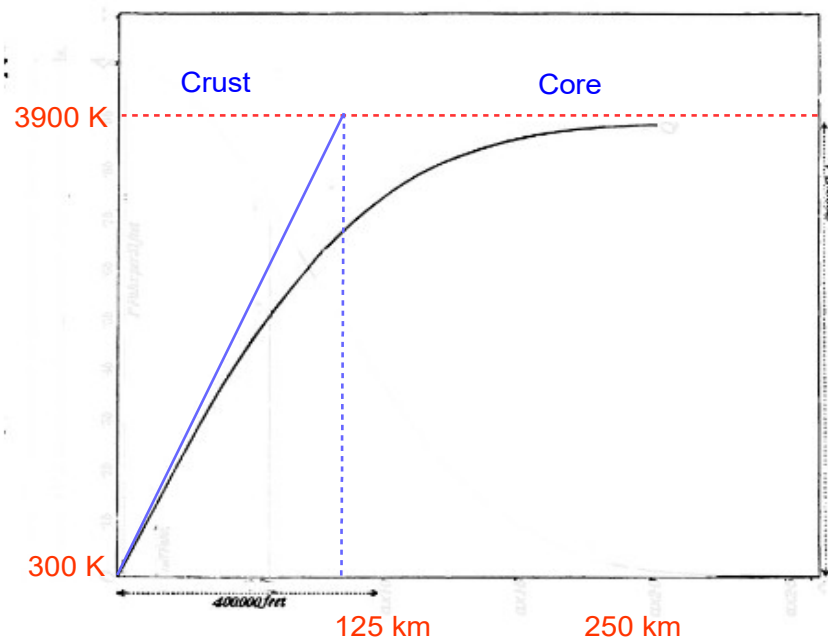
where T_0 is the initial temperature, k is the thermal conductivity of rock and β is the temperature gradient at the surface of the Earth⁴. This works out to be between 90 and 350 million years depending on the values taken for S and k for the material of the Earth's interior.

1 The reason for this is that when the Earth is hot, it cools very quickly. Most of the time taken to cool is therefore in the later stages of the process.

2 $S = 450$ J/kg/K, $\rho = 7900$ kg/m³, $\sigma = 5.7 \times 10^{-8}$ W/m/K, $R = 6.4 \times 10^6$ m, $T = 300$ K

3 This figure may be regarded as a baseline temperature and does not enter into his calculations.

4 $S = 700 - 1000$ J/kg/K, $\rho = 5500$ kg/m³, $k = 1.5 - 4$ W/m/K, $\beta = 0.04$ K/m, $T_0 = 3900$ K



Kelvin's graph of the Earth's current temperature profile

Looking back on his calculations, it may seem surprising that anyone took any notice of it at all, considering the huge assumptions that he made. In the first place, it was extremely unlikely that the Earth was totally uniform. Although Kelvin had no way of knowing what was in the Earth's interior, the evidence from volcanoes suggested that the Earth was molten beneath a solid crust, but that the immense pressure inside the Earth would render the rocks solid again below a certain depth. Secondly, it seemed unreasonable to assume that the surface temperature of the Earth is and always was 300K and that space had an unlimited capacity to absorb heat to keep it there – but without Stefan's law to guide him, there was no way he could factor in this aspect of the problem. In fact, the main reason why Kelvin was as confident of his results as he was was because a calculation based on the gravitational energy released by the gradual collapse of the Sun produced a very similar figure of 100 million years⁵. Taken together, these two independent estimates carried far more weight than either would on its own.

So where did Kelvin go so horribly wrong?

In the case of the Earth, the answer is not that he left out the effect of the heat generated by radioactivity. In fact the Earth receives 1000 times as much energy from the Sun every second as is generated by the decay of radioactive elements within it. On the other hand, in the case of the Sun, it was not radioactive decay but the process of nuclear fusion which was completely unknown to him and which is the reason why the Sun has been shining more or less constantly for 4,500 million years.

So what was Kelvin's mistake?

It is currently fashionable to credit an Irish engineer called John Perry with the honour of pointing out Kelvin's error. It was certainly Perry who first suggested the idea that the rocks of the Earth's mantle were only solid over short timescales and that they could flow like a liquid over millions of years. This meant that in addition to conduction, *convection* currents played a vital role in conveying heat from the Earth's interior to the surface. This observation completely destroyed

⁵ The Gravitational Potential Energy of a uniform sphere of mass M and Radius R is equal to $(-) 3GM^2 / 5R$. Using data readily available to Kelvin this works out to be about 2.3×10^{41} J. Now the Sun radiates about 4×10^{26} W of power; it could therefore shine for 5.8×10^{14} s = 20 million years. If we make the plausible assumption that the density of the Sun increases with depth, this estimate could easily be increased to 100 million years or more. The important thing is that the result is the same order of magnitude as Kelvin's estimate of the age of the Earth.

Kelvin's argument which was based entirely on the assumption that it was thermal *conduction* which was the dominant process. In 1895, Perry published a paper in *Nature* outlining a new argument which concluded that the Earth could be as much as 2 to 3 billion years old.

Now it is not obvious to see at first why increasing the rate of flow of heat from the interior could *increase* the calculated age of the Earth. Intuitively you would think that it would decrease it. But we must remember that Kelvin was using the *temperature gradient at the Earth's surface* as the defining measure, not the average temperature. In Perry's model, the Earth has a crust of rock approximately 100 km thick whose thermal conductivity is relatively low resting on an effectively liquid interior whose thermal conductivity may be regarded as being very high owing to its ability to transfer heat by convection. Like Kelvin, Perry assumed that, ever since it had formed, the temperature of the surface of the crust remained constant but his model required that the temperature of the core had cooled significantly during this period.

In other words, instead of Fourier's rounding of the temperature profile only involving heat contained in the top 5% of the Earth, Perry's model (represented by the blue lines on the graph) explains the reduction of the temperature gradient at the surface as due to the cooling of the whole of the Earth's core (i.e. the gradual lowering of the dotted red line). This is obviously going to take a lot longer.

I do not have access to Perry's original paper but a contemporary argument might have gone like this: knowing that the temperature gradient at the Earth's surface (β) was about 0.04 K/m and that the thermal conductivity of rock (k) was not more than 4 W/m/K, it follows that the Earth's centre is currently losing heat at a rate of not more than $kA\beta = 80$ TW where A is the surface area of the Earth ($1 \text{ TW} = 10^{12} \text{ W}$). (Compared with the energy absorbed at the Earth's surface from the Sun, this is quite a small figure⁶.) If we assume that the temperature gradient remains constant at 4 K per 100 m at greater depths then it would reach a temperature of 3900K at a depth of about 100 km. This is consistent with the idea that the Earth has a solid crust resting on a quasi-solid mantle. Let us suppose, with Perry, that below this depth, owing to convection currents, the Earth conductivity is effectively very large and that in consequence, we can assume that, below 100 km, the whole Earth is at a constant temperature. We can now argue as follows: Since the temperature difference between the surface and the core is $3900 - 300 = 3600 \text{ K}$, the power lost per unit temperature difference is $80 \text{ TW} / 3600 = 2.2 \times 10^{10} \text{ W/K}$. In the past, the temperature of the Earth's core may have been larger but once the crust has formed the total power lost from the core of the Earth will still be approximately equal to $(-2.2 \times 10^{10} (T_{\text{core}} - 300)) \text{ W}$.

Now we also have power lost $= MS dT_{\text{core}}/dt$ where M is the mass of the Earth and S is its specific heat capacity. Equating these and integrating we get:

$$T_{\text{core}} = 300 + 3600 e^{\frac{-2.2 \times 10^{10} t}{MS}} \quad (5)$$

Now the expression $MS / 2.2 \times 10^{10}$ has the dimensions of time and evaluates to $2.7 \times 10^{17} \text{ s}$ or 8.5 billion years. What this means is that the core temperature of the Earth would be expected to cool exponentially at a rate which would cool to one e^{th} (36%) of its original temperature in 8.5 By.

Perry's model does not define a single figure for the age of the Earth, it merely suggests that the timescale over which the Earth has cooled is to be measured in billions rather than millions of years.

Now although Perry was eventually proved correct, the idea that convection currents could exist in the Earth's mantle was fiercely resisted, not least by Kelvin himself who had 'proved' that the Earth had to be solid to account for the libations of the Moon. Perry's estimates of 2 to 3 billion years for the age of the Earth was also resisted by the geologists themselves who had spent three

⁶ This should be compared with the 60,000 TW received from the Sun. (see footnote 8 on page 5)

decades customising themselves to the idea that the Earth was not more than 100 million years old. In the end, it was not the discovery that the Earth contained an additional source of heat (from radioactive decay) that saw the end of Kelvin's timescale, it was the discovery of radiometric dating.

Radiometric dating

The phenomenon of radioactivity was discovered in the final decade of the nineteenth century and in a few years Rutherford and others had worked out all the basic principles. Atoms of a radioactive isotope such as the Uranium 238 decay at random times into other elements, eventually reaching a stable isotope of lead. The so-called half-life of this process is the time it takes for half of the original sample to decay and in the case of U_{238} this time is 4.5 billion years⁷. In 1907 an American scientist Bertram Boltwood measured the ratio of lead to uranium in a number of samples of rock and showed that some of them were over 2 billion years old. This was the first valid measurement of a lower bound for the age of the Earth. Subsequently rocks and meteorites have been found which are 4.5 billion years old. Since we believe that the whole solar system was formed at the same time, we believe that the Sun has been shining as a star for the same period of time. Knowing its mass and its energy output it is estimated that it will continue shining for another 5 billion years before ending its life as a red giant, swallowing up all the inner planets including Earth.

The final question

One question remains to be answered, however. Why was the interior of the Earth so hot in the first place?

In the footnote on page 3 we noted that the Gravitational Potential Energy of a sphere of mass M and radius R is $(-)\frac{3GM^2}{5R}$. If we suppose that, during the formation of the Earth all this energy is turned into heat, we have $\frac{3GM^2}{5R} = MST$ hence $T = \frac{3GM}{5RS}$. Using data appropriate to the Earth, we find that the temperature of the primordial Earth might have been as high as 37,000 K. It is more probable that the Earth was formed by the coalescing of small planetary bodies (called planetesimals) which had already cooled significantly so the temperature of the nascent Earth may never have been as high as this. It is therefore very difficult to say what length of time elapsed before the Earth cooled sufficiently to form a solid crust but undoubtedly this occurred at least 4.5 billion years ago. Assuming, for the sake of argument, that the mean temperature of the Earth's core is currently 3900K, we can use Perry's theory and equation (5) to estimate the temperature of the Earth's core 4.5 billion years ago. This is:

$$T_{core} = 300 + 3600e^{4.5/8.5} = 6400K \quad (6)$$

A perfectly reasonable estimate I would say.

Using this figure and equation (3) we can estimate the time taken for the Earth to cool from a very hot state to 6400 K by emitting radiation. This turns out to be only 8.5 years!⁸ In practice, the surface of the Earth will cool much more rapidly than the interior so what this is telling us is that, whatever the initial temperature of the primordial Earth, the crust started to form very early on in Earth's history and ever since then, the interior of the Earth has been cooling very slowly, thermally insulated from space by a thick blanket of solid rock of very low thermal conductivity.

7 If you are wondering how it is possible to measure such a long time in an experiment lasting a few minutes, you simply take a milligram of pure uranium which is known to contain 2.5×10^{18} atoms and using a Geiger counter measure how many alpha particles it is emitting per second. This will be about 12 decays per second. The half life is then given by the expression $\log_2 \times 2.5 \times 10^{18} / 12 = 1.4 \times 10^{17} \text{ s} = 4.5 \times 10^9 \text{ years}$.

8 $S = 1000 \text{ J/kg/K}$, $\rho = 5700 \text{ kg/m}^3$, $\sigma = 5.7 \times 10^{-8} \text{ W/m}^2\text{K}$, $R = 6.4 \times 10^6 \text{ m}$, $T = 6400 \text{ K}$

Conclusion

Kelvin knew that the temperature gradient at the Earth's surface (β) was about 0.04 K/m. He was also well aware that the thermal conductivity of rock (k) was not more than 4 W/m/K. He must have been aware, therefore, that the Earth is losing heat at no more than 80 TW. He must also have been aware that the Earth is receiving energy at a rate of 60,000 TW from the Sun⁹. The huge disparity between these figures indicates that the surface temperature of the Earth is far more dependant on the Sun's radiation than it is on the temperature of the Earth's core. In this respect, his (and Perry's) assumption that the Earth's crust had essentially remained at a constant temperature turned out to be perfectly justified. These figures also explain why Newton's estimate of the age of the Earth by calculating the rate at which a molten ball of rock would cool by radiation was totally invalid and why Kelvin never mentioned Stefan's law in respect of either the heat gained from the Sun or the heat radiated by the Earth into space.

It was his assumption that, since his astronomical observations had shown to his perfect satisfaction that the Earth was completely solid, thermal conduction was the only way heat could escape from the interior that was at fault. Perry's idea that the Earth's mantle could both be solid over short timescales but support convection currents over millions of years was largely ignored until Arthur Holmes championed Wegener's theory of continental drift¹⁰ in the 1930's.

As regards radioactivity, it is thought that of the 80 TW of heat lost from the core about 50% may be due to radioactivity. What this means is that 50% of the temperature gradient at the surface of the Earth due to cooling and 50% due to radioactivity. If Kelvin had known about this additional source of heat, he would have used a value of β of 0.02 instead of 0.04 K/m and his estimate of the age of the Earth would have increased by a factor of 4. This would have made very little difference to the subsequent debate.

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9 The first measurement of the solar constant was made by Claude Pouillet in 1838 who obtained a value of 1700 W/m² – a figure very close to the currently accepted value. Of this energy about 70% is immediately reflected into space so, on average, the Earth absorbs about 60,000 TW of heat from the Sun.

10 Wegener's theory was first published in 1912 but, like Perry, he was ignored.